BU-402-M

ABSTRACT

THE HISTORICAL DEVELOPMENT OF THE CONSTRUCTION OF HADAMARD MATRICES OF ORDER LESS THAN OR EQUAL TO 200

by

W. T. INKSTER and B. L. RAKTOE

University of Guelph and B. L. Raktoe

A Hadamard matrix (H-matrix) is a matrix consisting entirely of elements which are ± 1 and also possessing the orthogonal property $H \cdot H' = m I_m$. It was shown in 1893 by J. Hadamard that the maximum value of a determinant of order m, all elements of which are not greater than +1 in absolute value is $m^{m/2}$. Furthermore he showed that the upperbound of $m^{m/2}$ can be attained only if the determinant being considered is of an H-matrix. Thus if we construct a (-1,1) matrix which has a determinant of value $m^{m/2}$ we know that this matrix must be an H-matrix. It was shown by R.E.A.C. Paley in 1933 that for H-matrices of order m to exist it is necessary that m = 1, 2 or $m \equiv 0 \pmod 4$. Although the necessary condition is easily seen, the sufficiency of the existence of Hadamard matrices for all t = 1, 2, ..., where m = 1, 2, 4t has been conjectured by Paley but has never been proved. Empirical evidence, has so far, strongly supported this conjecture, but a valid proof still escapes all mathematicians.

In this paper we present the historical development of the construction of H-matrices of order $m \leq 200$ and also give constructed proofs. As the summary shows, the appendix lacks H_{188} because this is the only one which has defied construction ~~for~~ ~~any~~ as of today, in the range $m \leq 200$.

THE HISTORICAL DEVELOPMENT OF THE CONSTRUCTION OF HADAMARD MATRICES OF ORDER LESS THAN OR EQUAL TO 200

By W. T. INKSTER[1] and B. L. RAKTOE[2]

University of Guelph and Cornell University

BU-402-M December, 1971

1. INTRODUCTION Throughout the study of (0,1)-matrices (all elements of which are 0 or 1) we may, by a very simple transformation of $x \to 2x-1$, map any (0,1) matrix into a (-1,1) matrix (all elements of which are ± 1). When considering the class of all (-1,1) matrices one may study a very special type of (-1,1) matrices called "Hadamard Matrices".

A square matrix is considered to be orthogonal if the inner product between any two rows or columns equals zero. i.e., A is orthogonal if

$$(1.1) \qquad \sum_{j=0}^{m-1} A_{i_1 j} \cdot A_{i_2 j} = 0 \qquad (0 \le i \le m-1; \quad 0 \le j \le m-1) \quad i_1 \neq i_2$$

$$(1.2) \qquad \sum_{i=0}^{m-1} A_{i j_1} \cdot A_{i j_2} = 0 \quad j_1 \neq j_2$$

These orthogonal matrices in which all elements are ± 1 had been considered as long ago as 1867 by J. Sylvester [30].

---

1. Submitted as a Project for M.S. degree, Dept. of Mathematics and Statistics, University of Guelph.
2. Visiting Professor, Biometrics Unit, Cornell University.

A Hadamard matrix (H-matrix) is a matrix consisting entirely of elements which are ± 1 and also possessing the orthogonal property as first considered by Sylvester. Thus a square $(-1,1)$ matrix H of order m is an H-matrix if $H \cdot H^T = mI_m$ where H^T is the transpose of H and I_m is the identity matrix of order m. It has also been shown in 1893 by J. Hadamard [14] that the maximum value of a determinant of order m, all elements of which are not greater than +1 in absolute value, is $m^{m/2}$. Furthermore it was also shown that the upper bound of $m^{m/2}$ can be attained only if the determinant being considered is of an H-matrix. Thus if we construct a $(-1,1)$ matrix which has a determinant of value $m^{m/2}$ we know this matrix must be a Hadamard matrix.

It has been shown by R. E. A. C. Paley [23] in 1933 that for H-matrices of order m to exist it is necessary that $m = 1,2$, or $m \equiv 0 \pmod 4$. Although the necessary condition is easily seen, the sufficiency of the existence of Hadamard matrices for all $t = 1,2,\cdots\infty$ where $m = 1,2,4t$ has been conjectured by Paley but has never been proven. Empirical evidence, has sofar, strongly supported this conjecture but a valid proof still escapes all mathematicians.

To see how $m > 2$ must be a multiple of 4, consider the first three rows of any normalized H-matrix. In general we have x columns of the type $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$; y columns of type $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$; z columns of type $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and w columns of type $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$.

Thus we have four equations

(1.3) $x + y + z + w = m$

$x + y - z - w = 0$ (Row 1 orthogonal to Row 2)

$x - y + z - w = 0$ (Row 1 orthogonal to Row 3)

$x - y - z + w = 0$ (Row 2 orthogonal to Row 3)

Solving we have $x = y = z = w = m/4$ and therefore m must be a multiple of 4 if H is an H-matrix of order $m > 2$.

In this paper we will present some of the possible methods of construction of H-matrices of order $m \leq 200$ and also give constructed proof of these matrices.

The importance of Hadamard matrices can be seen in many applied combinatorial problems as well as being extremely handy in matrix inversion using Hadamard's maximum determinant property. J. Todd [32] has shown that the existence of a normalized H-matrix of order 4t is equivalent to the existence of a symmetrical incomplete block design with parameters $v = b = 4t-1$; $r = k = 2t-1$; $\lambda = t-1$ (rows and columns are numbered $0,1,\cdots 4t-1$). M. Hall, Jr. [15,16], shows how from a normalized H-matrix we may construct a symmetric block design and vice-versa. If we say $a_i \in B_j$ if $b_{ij} = +1$ in $H = (b_{ij})$ and $a_i \notin B_j$ if $b_{ij} = -1$, $(i,j > 0)$, we have constructed an incidence system of $v = 4t-1$ blocks and v objects with an incidence matrix $A = (a_{ij})$. Similarly given an incidence matrix A of a symmetric design with $v = 4t-1$; $k = 2t-1$; $\lambda = t-1$, we can construct an H-matrix $H = (b_{ij})$, $i,j = 0,\cdots 4t-1$ by setting

$$(1.4) \qquad \left[\begin{array}{llll} b_{0j} = b_{i0} = 1 & & & i,j = 0,\cdots 4t-1 \\ b_{ij} = 1 & \text{if} & a_i \in B_j & i,j > 0 \\ b_{ij} = -1 & \text{if} & a_i \notin B_j & i,j > 0 \end{array}\right]$$

For example consider:

$$H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \quad ; \quad t = 2$$

From H_8 we can construct a symmetric block design where:

$b = 4t-1 = 7 = \#$ of blocks

$k = 2t-1 = 3 = \#$ of objects per block

$v = 4t-1 = 7 =$ total $\#$ of objects

$\lambda = t-1 = 1 = \#$ of blocks containing any distinct pair $(a_i a_j)$

Thus

$$\begin{aligned} B_1 &= [a_2, a_4, a_6] \\ B_2 &= [a_1, a_4, a_5] \\ B_3 &= [a_3, a_4, a_7] \\ B_4 &= [a_1, a_2, a_3] \\ B_5 &= [a_2, a_5, a_7] \\ B_6 &= [a_1, a_6, a_7] \\ B_7 &= [a_3, a_5, a_6] \ . \end{aligned}$$

One may also see from Bose and Shrikhande [28] how a connection has been established between Hadamard matrices and the maximal binary codes $M(4t, 2t; 8t)$; $M(4t-1, 2t; 8t)$ and $M(4t-2, 2t; 2t)$ where $M(n, d; m)$ imply a set of mn-place sequences with 0 and 1 such that the Hamming distance between any two sequences is equal to or greater than d.

Recently B. Raktoe and W. Federer [25] have, in the context of main effect fractional replicates of the 2^n factorial, shown that all theory available for semi-normalized (-1,1) matrices [1st column contains all +1's] is applicable to semi-normalized (0,1) matrices and vice-versa. We also see from [25] how in the class of all semi-normalized square (0,1) matrices (of order m+1) the absolute value of the determinant of a matrix in this class is maximized at $2^{-m}\ (m+1)^{\frac{1}{2}(m+1)}$ when the (0,1) matrix is obtained from a semi-normalized H-matrix by setting all -1's equal to zero, thus resulting in an optimal main effect plan as discussed by Plackett and Burman earlier [24].

Hotelling [18], Kishen [21], Banerjee [1,2,3], and Mood [22], along with Plackett and Burman [24] illustrate precisely the degree of importance Hadamard matrices play in choosing optimum experimental designs in weighing problems, particularly for chemical balances. More recently Hedayat and Shrikhande (Sankhyā, Series A, Vol. 33, (1971)) have exhibited the fact that from a set of r-2 mutually orthogonal latin squares of order 2r one may obtain a Hadamard matrix of order $4r^2$.

At times throughout the history of Hadamard matrices we speak of "skew-type" Hadamard matrices or Hadamard matrices of "type I". An H-matrix of order m is considered to be "skew-type" if $H = S+I_m$ where $S^T = -S$. In particular $SS^T = -S^2 = (m-1)I_m$. Skew-type H-matrices have applications in tournaments [31]; the theory of finite projective planes [19]; as well as in the construction of H-matrices of certain orders.

2. CONSTRUCTION OF HADAMARD MATRICES

Earliest constructions of Hadamard matrices date back to 1867 when J. Sylvester [30] developed a precise method of construction for H-matrices of order 2^r, $r = 1,2,\cdots$.

Thirty-one years later in 1898 U. Scarpis [27] proved the existence of H-matrices of order $m = p(p+1)$ where $p \equiv 3 \pmod 4$, p an odd prime.

For the remainder of the 19^{th} century and the first thirty-three years of the 20^{th} century nothing was developed in Hadamard matrices. However, during the year of 1933 R. Paley [23] made perhaps the greatest contribution to the construction of H-matrices. By the end of Paley's paper the existence and construction of H-matrices of order m lacked only for the six orders m = 92, 116, 156, 172, 184 and 188 when $m = 1,2,4t$, $t = 1,2,\cdots 50$.

The construction and proof of H-matrices are heavily dependent upon the direct product (Kronecker product) of 2 matrices. If $A = (a_{ij})$ is a matrix of order m and $B = (b_{rs})$ is a matrix of order ℓ, then the direct product $A \times B$ results in a square matrix C of order $m\ell$.

$$A \times B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1m}B \\ a_{21}B & a_{22}B & \cdots & a_{2m}B \\ & & \ddots & \\ a_{m1}B & a_{m2}B & \cdots & a_{mm}B \end{bmatrix} \tag{2.1}$$

Paley's proof's of existence and construction of H-matrices may be summarized by the following theorems as found in his paper [23].

Theorem 2.1 If we have an H-matrix of order m and an H-matrix of order ℓ, then we can construct an H-matrix of order $m\ell$ by taking the direct product of the two-matrices.

Using the properties $(A \times B)\cdot(C \times D) = AC \times BD$ and also $(A \times B)^T = A^T \times B^T$ the proof is very straightforward. From theorem 2.1 we have now constructed H-matrices of orders m = 1, 2, 4, 8, 16, 32, 64, 128 (considering only $m \leq 200$).

For a simple example consider $H_2 \times H_2 = H_4 =$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Theorem 2.2 Let m be of the form p+1, where $p \equiv 3 \pmod 4$, p prime. Then we can construct an H-matrix of order m.

Constructions using this theorem and others to follow are based upon the Legendre symbol $\chi\left(\frac{j-i}{p}\right)$ involving quadratic residues (mod p), where i and j are the row and column numbers respectively and will always be considered to be numbered from 0 to 4t-1 .

(2.2) $\chi\left(\frac{j-i}{p}\right) = 1$ if there exists an integer solution to $x^2 \equiv (j-i) \pmod p$

$\chi\left(\frac{j-i}{p}\right) = -1$ if no integer solution exists

Theorem 2.1 and 2.2 both lead to normalized H-matrices and an H-matrix using (2.2) may be defined as follows:

(2.3)

$H(i,j) = 1$ $(i = 0 \text{ or } j = 0)$

$H(i,i) = -1$ $(1 \le i \le p)$

$H(i,j) = \chi\left(\frac{j-i}{p}\right)$ $(1 \le i \le p,\ 1 \le j \le p)$
$i \neq j$

When using Legendre symbols in the construction, one important property to remember is that

$$\chi\left(\frac{j-i}{p}\right) = \chi\left(\frac{-1}{p}\right)\chi\left(\frac{i-j}{p}\right) \tag{2.4}$$

Since $\chi(-1/p) = -1$ when $p \equiv 3 \pmod 4$, p prime then for all constructions using Theorem (2.2) we may use the fact $\chi\left(\frac{j-i}{p}\right) = -\chi\left(\frac{i-j}{p}\right)$.

For example consider the case when $m = 12 = (11+1) = (p+1)$

$\chi\left(\frac{1}{p}\right) = +1$ since an integer solution exists for $x^2 \equiv 1 \pmod{11}$.

i.e., an integer solution of $x = 10$ satisfies $x^2 = p\cdot K + 1 = 11\cdot K + 1$ when $K = 9$.

Similarly,	$\chi(2/p) = -1$	and thus	$\chi(-2/p) = +1$
	$\chi(3/p) = +1$		$\chi(-3/p) = -1$
	$\chi(4/p) = +1$		$\chi(-4/p) = -1$
	$\chi(5/p) = +1$		$\chi(-5/p) = -1$
	$\chi(6/p) = -1$		$\chi(-6/p) = +1$
	$\chi(7/p) = -1$		$\chi(-7/p) = +1$
	$\chi(8/p) = -1$		$\chi(-8/p) = +1$
	$\chi(9/p) = +1$		$\chi(-9/p) = -1$
	$\chi(10/p) = -1$		$\chi(-10/p) = +1$

Thus

$$H_{12} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

where $H_{12} \cdot H_{12}^T = 12I_{12}$.

Constructions based on Theorem (2.2) and future theorems to be discussed have been calculated using an I.B.M. computer at the University of Guelph and are printed in the appendix.

Using Theorem (2.1) and Theorem (2.2) we have now added H-matrices of order m = 12, (24, 48, 96, 192), 20, (40, 80, 160), 44, (88, 176), 60 (120), 68, (136), 84, (168), 108, 132, 140, and 168. The numbers not in brackets are the orders of H-matrices constructed by Theorem (2.2) and the following numbers within brackets are the orders of H-matrices constructed by Theorem (2.1) and the new H-matrices just constructed. For example, $H_{24} = H_2 \times H_{12}$.

<u>Theorem 2.3</u> Let m be divisible by 4 and of the form $m = 2^K(p+1)$, p prime, then we can construct an H-matrix of order m.

When $p = 2$ or $p \equiv 3 \pmod 4$ the proof follows immediately from theorems (2.1) and (2.2). When $p \equiv 1 \pmod 4$, p prime and $k = 1$ the following construction is given.

$$\begin{array}{ll} & H(2i,0) = H(2i,1) = H(2i+1,0) = -H(2i+1,1) = 1 \quad (1 \le i \le p) \\ (2.5) & H(0,2i) = H(0,2i+1) = H(1,2i) = -H(1,2i+1) = 1 \quad (1 \le i \le p) \\ & H(2i,2i) = -H(2i,2i+1) = -H(2i+1,2i) = -H(2i+1,2i+1) = 1 \quad (0 \le i \le p) \\ & H(2i,2j) = H(2i,2j+1) = H(2i+1,2j) = -H(2i+1,2j+1) = \chi\left(\frac{j-i}{p}\right) \\ & \qquad (1 \le i \le p,\ 1 \le j \le p \quad i \ne j) \end{array}$$

In constructions of this type, since

$\chi(-1/p) = +1$ when $p \equiv 1 \pmod 4$, p prime we may use the fact $\chi\left(\frac{j-i}{p}\right) = \chi\left(\frac{i-j}{p}\right)$.

For example, consider $m = 36 = 2^1(17+1)$ where $K = 1$, $p = 17$.

$\chi(1/p) = \chi(-1/p) = +1$ since an integer solution of $x = 1$ solves $x^2 = 17 \cdot K+1$ when $K = 0$ and $x = 4$ solves $x^2 = 17 \cdot K-1$ when $K = 1$.

$$\begin{aligned} \chi(2/p) &= \chi(-2/p) = 1 \\ \chi(3/p) &= \chi(-3/p) = -1 \\ \chi(4/p) &= \chi(-4/p) = 1 \\ \chi(5/p) &= \chi(-5/p) = -1 \\ \chi(6/p) &= \chi(-6/p) = -1 \\ \chi(7/p) &= \chi(-7/p) = -1 \\ \chi(8/p) &= \chi(-8/p) = 1 \\ \chi(9/p) &= \chi(-9/p) = 1 \\ \chi(10/p) &= \chi(-10/p) = -1 \\ \chi(11/p) &= \chi(-11/p) = -1 \end{aligned}$$

$$\chi(12/p) = \chi(-12/p) = -1$$
$$\chi(13/p) = \chi(-13/p) = 1$$
$$\chi(14/p) = \chi(-14/p) = -1$$
$$\chi(15/p) = \chi(-15/p) = 1$$
$$\chi(16/p) = \chi(-16/p) = 1$$

In this construction $(j-i)$ takes on values $\pm1, \pm2, \cdots \pm(p-1)$. For the actual H-matrix see appendix for H-matrix of order 36.

With this construction and Theorem (2.1) we may now add H-matrices of order m = 36, (72,144), 76, (152), 124, 148, 180, and 196. Once again values of m inside brackets represent H-matrices of order m constructed by Theorem (2.1).

<u>Theorem 2.4</u> Let m be divisible by 4 and of the form $2^K(p^h+1)$, p an odd prime. Then we can construct an H-matrix of order m.

The construction based on this theorem is dependent upon quadratic residues in the Galois field of polynomials (mod p, mod P(x)) where P(x) is an irreducible polynomial of degree h , [11]. If K = 0 and $p^h \equiv 3 \pmod 4$ then our H-matrix may be defined as:

(2.6)
$$H(i,j) = 1 \qquad (i = 0,\ j = 0)$$
$$H(i,j) = \chi\left(\frac{\xi_j - \xi_i}{p}\right) \qquad (1 \le i \le p^h,\ 1 \le j \le p^h,\ i \ne j)$$
$$H(i,i) = -1 \qquad (1 \le i \le p^h)$$

where $\xi_1, \xi_2, \cdots \xi_{p^h}$ denote the marks of the field arranged in any order.

For example if $m = 28 = 3^3+1$ and $P(x) = x^3+2x+1$ $(h = 3,\ p = 3)$ the marks of the field in any order $(\xi_1, \xi_2, \cdots \xi_{27})$ are

$\{0,\ 1,\ 2,\ x,\ 2x,\ x+1,\ x+2,\ 2x+1,\ 2x+2,\ x^2,\ 2x^2,\ x^2+x,\ x^2+2x,\ 2x^2+x,\ 2x^2+2x,$
$x^2+1,\ x^2+2,\ 2x^2+1,\ 2x^2+2,\ x^2+x+1,\ x^2+x+2,\ x^2+2x+1,\ x^2+2x+2,\ 2x^2+x+1,$
$2x^2+x+2,\ 2x^2+2x+1,\ 2x^2+2x+2\}$

Thus from (2.6) we get the following Hadamard matrix of order 28.

HADAMARD MATRIX OF ORDER 28

$$\left[\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrr}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&-1&1&-1&-1&-1&-1&1&1&1&1&-1&1&1&1&-1&1&1&-1&-1&-1&1&-1&1&-1&-1&1&-1\\
1&-1&-1&1&-1&-1&-1&1&1&1&1&1&-1&-1&1&1&-1&1&1&1&-1&-1&-1&-1&1&-1&-1&1\\
1&1&-1&-1&-1&-1&-1&1&1&1&-1&1&1&1&-1&1&1&-1&1&-1&1&-1&1&-1&-1&1&-1&-1\\
1&1&1&1&-1&1&-1&-1&-1&-1&1&1&-1&1&-1&1&1&1&-1&-1&1&-1&-1&-1&1&-1&1&-1\\
1&1&1&1&-1&-1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&1&1&-1&-1&1&1&-1&-1&-1&-1&1\\
1&1&1&1&1&-1&-1&-1&-1&-1&1&-1&1&-1&1&1&1&-1&1&1&-1&-1&-1&1&-1&1&-1&-1\\
1&-1&-1&-1&1&1&1&-1&1&-1&1&1&-1&1&1&-1&1&-1&1&-1&1&-1&-1&1&-1&-1&-1&1\\
1&-1&-1&-1&1&1&1&-1&-1&1&-1&1&1&-1&1&1&1&1&-1&-1&-1&1&-1&-1&1&1&-1&-1\\
1&-1&-1&-1&1&1&1&1&-1&-1&1&-1&1&1&-1&1&-1&1&1&1&-1&-1&1&-1&-1&-1&1&-1\\
1&-1&-1&1&-1&1&-1&-1&1&-1&-1&1&-1&-1&-1&-1&1&1&1&1&-1&1&1&1&-1&1&1&-1\\
1&1&-1&-1&-1&-1&1&-1&-1&1&-1&-1&1&-1&-1&-1&1&1&1&1&1&-1&-1&1&1&-1&1&1\\
1&-1&1&-1&1&-1&-1&1&-1&-1&1&-1&-1&-1&-1&-1&1&1&1&-1&1&1&1&-1&1&1&-1&1\\
1&-1&1&-1&-1&-1&1&-1&1&-1&1&1&1&-1&1&-1&-1&-1&-1&1&1&-1&1&-1&1&1&1&-1\\
1&-1&-1&1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&1&1\\
1&1&-1&-1&-1&1&-1&1&-1&-1&1&1&1&1&-1&-1&-1&-1&-1&1&-1&1&-1&1&1&1&-1&1\\
1&-1&1&-1&-1&1&-1&-1&-1&1&-1&-1&-1&1&1&1&-1&1&-1&1&1&-1&1&1&-1&1&-1&1\\
1&-1&-1&1&-1&-1&1&1&-1&-1&-1&-1&-1&1&1&1&-1&-1&1&-1&1&1&-1&1&1&1&1&-1\\
1&1&-1&-1&1&-1&-1&-1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&1&-1&1&1&-1&1&-1&1&1\\
1&1&-1&1&1&1&-1&1&1&-1&-1&-1&1&-1&1&-1&-1&1&-1&-1&1&-1&-1&-1&-1&1&1&1\\
1&1&1&-1&-1&1&1&-1&1&1&1&-1&-1&-1&-1&1&-1&-1&1&-1&-1&1&-1&-1&-1&1&1&1\\
1&-1&1&1&1&-1&1&1&-1&1&-1&1&-1&1&-1&-1&1&-1&-1&1&-1&-1&-1&-1&-1&1&1&1\\
1&1&1&-1&1&-1&1&1&1&-1&-1&1&-1&-1&-1&1&-1&1&-1&1&1&1&-1&1&-1&-1&-1&-1\\
1&-1&1&1&1&1&-1&-1&1&1&-1&-1&1&1&-1&-1&-1&-1&1&1&1&1&-1&-1&1&-1&-1&-1\\
1&1&-1&1&-1&1&1&1&-1&1&1&-1&-1&-1&1&-1&1&-1&-1&1&1&1&1&-1&-1&-1&-1&-1\\
1&1&1&-1&1&1&-1&1&-1&1&-1&1&-1&-1&1&-1&-1&-1&1&-1&-1&-1&1&1&1&-1&1&-1\\
1&-1&1&1&-1&1&1&1&1&-1&-1&-1&1&-1&-1&1&1&-1&-1&-1&-1&-1&1&1&1&-1&-1&1\\
1&1&-1&1&1&-1&1&-1&1&1&1&-1&-1&1&-1&-1&-1&1&-1&-1&-1&-1&1&1&1&1&-1&-1
\end{array}\right]$$

Now we are able to add to our list of H-matrices constructions for $m = 28$, $(56,112)$, 52, (104), and 100, (200).

Paley also extends a result due to Scarpis [27] to show the existence of H-matrices of order m of the form $2^K p(p+1)$, $p \equiv 3 \pmod 4$ and p prime. However, this result is not required for $m \le 200$ since the next matrix of this type not already covered is $2^0(43)(43+1) = 1892$.

Now we may summarize our results until the end of 1933 with the following statement. If $m \le 200$, is divisible by 4, we may construct an H-matrix except possibly in the cases of m = 92, 116, 156, 172, 184, and 188. The different cases may be tabulated in the following table (page 317, [23]) where only one of the possible solutions is given:

TABLE 1

$m = 4 = 2^2$	$72 = 2^2(17+1)$	$140 = 139+1$
$8 = 2^3$	$76 = 2(37+1)$	$144 = 2^3(17+1)$
$12 = 11+1$	$80 = 2^2(19+1)$	$148 = 2(73+1)$
$16 = 2^4$	$84 = 83+1$	$152 = 2^2(37+1)$
$20 = 19+1$	$88 = 2(43+1)$	$156 = ?$
$24 = 2(11+1)$	$92 = ?$	$160 = 2^3(19+1)$
$28 = 3^3+1$	$96 = 2^3(11+1)$	$164 = 163+1$
$32 = 2^5$	$100 = 2(7^2+1)$	$168 = 2(83+1)$
$36 = 2(17+1)$	$104 = 2^2(5^2+1)$	$172 = ?$
$40 = 2(19+1)$	$108 = 107+1$	$176 = 2^2(43+1)$
$44 = 43+1$	$112 = 2^2(3^3+1)$	$180 = 2(89+1)$
$48 = 2^2(11+1)$	$116 = ?$	$184 = ?$
$52 = 2(5^2+1)$	$120 = 2(59+1)$	$188 = ?$
$56 = 2(3^3+1)$	$124 = 2(61+1)$	$192 = 2(11+1)$
$60 = 59+1$	$128 = 2^7$	$196 = 2(97+1)$
$64 = 2^6$	$132 = 131+1$	$200 = 2^2(7^2+1)$
$68 = 67+1$	$136 = 2^2(37+1)$	

For the next eleven years nothing of any significance developed in the theory of Hadamard matrices. However, in 1944, John Williamson [35] presented his ideas and work which eventually led to the remaining unsolved constructions as well as additional methods of constructing different H-matrices of orders already proven to exist.

By 1944 it was known that a Hadamard matrix of order m exists when:

(i) $m = 2$ [23]

(ii) $m = p^h+1 \equiv 0 \pmod 4$, p odd prime [23]

(iii) $m = 2^K(p^h+1)$, p odd prime [23]

(iv) $m = p(p+1)$, $p \equiv 3 \pmod 4$, p odd prime [27]

Since a direct product of two H-matrices is an H-matrix [23], an H-matrix exists of any order which is a product of factors of types (i), (ii), (iii), or (iv).

In 1944 John Williamson [35] adds:

Theorem 2.5 If there exists an H-matrix of order $m > 1$ then there exists an H-matrix of order $m(p^h+1)$ where p is any odd prime.

Theorem 2.6 There exists an H-matrix of order $N(N-1)$ where $N = 2^t k_1 k_2 \cdots k_r$ where $k_i = p_i^{h_i}+1$ and p_i is an odd prime such that $p_i^{h_i}+1 \equiv 0 \pmod 4$.

However, the most important part of his paper investigates a connection between special H-matrices of order 4n, the n^{th} roots of unity and the representation of 4n as the sum of the squares of four integers. With these interrelations Williamson was able to add a construction for a Hadamard matrix of order 172.

To employ Williamson's method we shall consider four, square symmetric, Hermitan matrices A, B, C, D, each of which is of the order m. If these matrices satisfy the equation:

$$A^2 + B^2 + C^2 + D^2 = 4mI_m \tag{2.7}$$

then

$$H = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix} \tag{2.8}$$

is an H-matrix of order 4m.

From the following example we can see that matrices of this type do exist.

If $m = 3$ and

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} ; \quad B = C = D = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

then (2.8) is an H-matrix of order 12 since

$$A^2 + B^2 + C^2 + D^2 = 12I_3 = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

The problem remaining lies in constructing the matrices A, B, C, and D.

Consider a matrix W of order m corresponding to a cyclic permutation of order m where

$$(2.9) \qquad W = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ & & & \ddots & & \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad ; \quad W^m = I_m$$

A, B, C, D will commute with each other if we define them as polynomials in W such that

$$(2.10) \qquad \begin{aligned} A &= a_0 I + \sum_{i=1}^{m-1} a_i W^i \\ B &= b_0 I + \sum_{i=1}^{m-1} b_i W^i \\ C &= c_0 I + \sum_{i=1}^{m-1} c_i W^i \\ D &= d_0 I + \sum_{i=1}^{m-1} d_i W^i \end{aligned}$$

Since $W^T = W^{-1}$, the matrices A, B, C, D will be symmetric if $a_{n-i} = a_i$; $b_{n-i} = b_i$; $c_{n-i} = c_i$ and $d_{n-i} = d_i$.

From this point on we will assume m is odd and $a_0 = b_0 = c_0 = d_0 = 1$. From these assumptions an important theorem results which will aid as a check in the construction of H-matrices and also is very important in the theoretical development of the constructions.

<u>Theorem 2.7</u> If m is odd, and if the matrices A, B, C, and D of (2.10) are chosen with signs so that $a_0 = b_0 = c_0 = d_0 = 1$ and satisfy (2.7), then for each $i = 1, \cdots m-1$ of the a_i, b_i, c_i, d_i exactly three are of the same sign.

By considering the group ring $R(G)$ over the integers where G is the cyclic group $1, w, w^2, \cdots w^{m-1}$; $w^m = 1$; w corresponding to W we may define

(2.11) $$A = \sum_{i=0}^{m-1} a_i w^i$$ where w is any n^{th}-root of unity and similarly for B, C, D.

Also define:

(2.12)
$$\begin{aligned} 2T_1 &= A + B + C - D \\ 2T_2 &= A + B - C + D \\ 2T_3 &= A - B + C + D \\ 2T_4 &= -A + B + C + D \end{aligned}$$

Thus,

(2.13) $$T_1^2 + T_2^2 + T_3^2 + T_4^2 = 4m \ .$$

From theorem (2.7) each $T_i = 1 \pm 2w^j \pm \cdots 2w^K$. For each $j = 1, 2, \cdots m-1$ we have a term $\pm\, 2w^j$ in exactly one and only one of the T's.

(2.12) can be alternately expressed as

(2.14)
$$\begin{aligned} 2A &= T_1 + T_2 + T_3 - T_4 \\ 2B &= T_1 + T_2 - T_3 + T_4 \\ 2C &= T_1 - T_2 + T_3 + T_4 \\ 2D &= -T_1 + T_2 + T_3 + T_4 \end{aligned}$$

By the theorem of Lagrange when m is odd we may express 4m as a sum of four integers. Using this fact and considering the particular n^{th} root of unity when $w = 1$ we may now utilize Williamson's method to construct certain Hadamard matrices. To help understand his method consider the following examples:

If $m = 3$,

$$4m = 12 = 1^2 + 1^2 + 1^2 + 3^2 = 1^2 + 1^2 + 1^2 + (1-2w_1)^2$$

where $w_i = w^i + w^{n-i}$

If $m = 5$ we may express 4m as

$$\begin{aligned} 20 &= 1^2 + 1^2 + 3^2 + 3^2 = 1^2 + 1^2 + (1-2w_1)^2 + (1-2w_2)^2 \\ &= T_1^2 + T_2^2 + T_3^2 + T_4^2 \\ &= 20 \end{aligned}$$

Thus,
$$\begin{aligned} T_1 &= 1 \\ T_2 &= 1 \\ T_3 &= 1-2w_1 = 1-2w-2w^4 \\ T_4 &= 1-2w_2 = 1-2w^2-2w^3 \end{aligned}$$

Using (2.14) we now get

$$\begin{aligned} A &= 1 - w + w^2 + w^3 - w^4 \\ B &= 1 + w - w^2 - w^3 + w^4 \\ C &= 1 - w - w^2 - w^3 - w^4 \\ D &= 1 - w - w^2 - w^3 - w^4 \end{aligned}$$

Thus from (2.11) this implies

$$a_0 = b_0 = c_0 = d_0 = 1$$
$$-a_1 = a_2 = a_3 = -a_4 = 1$$
$$b_1 = -b_2 = -b_3 = b_4 = 1$$
$$c_1 = c_2 = c_3 = c_4 = d_1 = d_2 = d_3 = d_4 = -1$$

Now that we know the coefficients a_i, b_i, c_i and d_i using (2.10)

$$A = 1 \cdot I_5 - \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

Similarly,

$$B = \begin{bmatrix} 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 \end{bmatrix} ; \; C = D = \begin{bmatrix} 1 & -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & 1 \end{bmatrix}$$

and hence an H-matrix of order 20 is constructed from (2.8).

For further representations see pages 78 and 79 of [35].

The most important representation of $4n$ as the sum of squares worked out by Williamson is for $n = 43$.

$$172 = 1^2 + 1^2 + 1^2 + 13^2 = (1+2n_0-2n_2)^2 + (1+2n_3-2n_1)^2$$
$$+ (1+2n_4-2n_6)^2 + (1+2n_5)^2$$

where $n_j = w_{3^j} + w_{3^{j+7}} + w_{3^{j+14}} \qquad j = 0,1,2,3,4,5,6$.

Now we have added an H-matrix of order 172 to our list of known constructions. Due to the laborious computation involved, Williamson was unable to represent any of the remaining unknown orders as a sum of four squares. However, other people have been more successful with the aid of modern computors.

Although J. Williamson made extremely significant contributions to Hadamard matrix theory, his publications were limited. In 1947, following his papers in 1944 [35] and 1946 [36, 37]; Williamson shows in [38] that an H-matrix of order m also exists when:

(1) $m = q(q+3)$, where q and q+4 are both products of thypes $m = 2$ and $m = p^h+1 \equiv 0 \pmod 4$, p prime.

(2) $m = m_1 m_2 (p^h+1)p^h$, where $m_1 > 1$ and $m_2 > 1$ are order of H-matrices, p odd prime.

(3) $m = m_1 m_2 m_3 (m_3+3)$, where $m_1 > 1$ and $m_2 > 1$ are orders of H matrices and m_3 and m_3+4 are both of the form p^h+1, p odd prime.

As a result, the existince of H-matrices of order m can be increased for $m = (56)(59) = 2(3^3+1)(59)$ and $m = 4(73)(74)$ and $m = 4(230)(233)$.

Although this paper is particularly concerned with the history of the construction of Hadamard matrices of order $m \le 200$, references will be noted for additional material relevant to other aspects of Hadamard matrices of all orders.

In 1953, A. Brauer [8] discusses a new class of H-matrices and in 1959 Bose and Shrikhande [28] establsihes a connection between H-matrices and binary codes and balanced incomplete block designs.

During this same year E. Dade and K. Goldberg [10] present another interesting method of construction of H-matrices. If there exists a (0,1) matrix A of order 4n-1 satisfying $AA^T = nI + (n-1)J$ where I is the identity matrix and J is a matrix of order 4n-1 consisting entirely of 1's, then there exists a normalized H-matrix H where

$$H = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & & & & \\ \vdots & & 2A-J & & \\ 1 & & & & \end{bmatrix}$$

For the proof of this construction one uses a transitive permutation group of degree $4n-1$ and odd order whose subgroups leaving one element fixed have the transitivity sets each.

Further developments and constructions of generalized Hadamard matrices can be read in A. Butson's paper [9] published lin 1962.

It was during this same year 1962, 18 years after Williamson's important paper representing $4m$ as a sum of four squares, that L. Baumert, W. G. Golomb and M. Hall, Jr. were successful in doscovering a construction of an H-matrix of order 92 utilizing Williamson's method [4]. Now with the existence of an H-matrix for $m = 92$ and thus $m = 2(92) = 184$ only three remaining orders ≤ 200 were unsolved, i.e., $m = 116$, 156 and 188.

In 1965, further constructions have also been discussed by H. Ehlich [12]. However, a more important result also developed in 1965, for L. Baumert and M. Hall, Jr. continued to solve the unknown constructions [5]. Once again this construction for an H-matrix of order 156 is based upon Williamson's method. However, this construction has an added feature. A Hadamard matrix H is of the Williamson type if

$$H = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix}$$

where A,B,C,D is a symmetric circulant square matrix. However, for $m = 156 = 12(13)$ Baumert and Hall have constructed symmetric circulant matrices A,B,C,D, of order 13 where

$$H = \begin{bmatrix} A & A & A & B & -B & C & -C & -D & B & C & -D & -D \\ A & -A & B & -A & -B & -D & D & -C & -B & -D & -C & -C \\ A & -B & -A & A & -D & D & -B & B & -C & -D & C & -C \\ B & A & -A & -A & D & D & D & C & C & -B & -B & -C \\ B & -D & D & D & A & A & A & C & -C & B & -C & B \\ B & C & -D & D & A & -A & C & -A & -D & C & B & -B \\ D & -C & B & -B & A & -C & -A & A & B & C & D & -D \\ -C & -D & -C & -D & C & A & -A & -A & -D & B & -B & -B \\ D & -C & -B & -B & -B & C & C & -D & A & A & A & D \\ -D & -B & C & C & C & B & B & -D & A & -A & D & -A \\ C & -B & -C & C & D & -B & -D & -B & A & -D & -A & A \\ -C & -D & -D & C & -C & -B & B & B & D & A & -A & -A \end{bmatrix}$$

One year later, in 1966, L. D. Baumert constructed still another H-matrix of order $m = 116$ [6]. This construction is of the Williamson type where A,B,C,D are square symmetric circulant matrices of order 29. Now there remains only one unsolved case less than 200, that being an H-matrix of order 188.

In 1966, K. Goldberg [13] presented another existence theorem by proving if there exists an H-matrix of type 1 (skew-type) and order h then there also exists an H-matrix of type 1 and order $(h-1)^3+1$.

By 1967, Hadamard matrices had been shown to exist for all orders $4m \leq 400$ except 188, 236, 260, 268, 292, 356, 372 and 376. During this year E. Spence [29] added another set of values by proving the following theorems:

(1) If the primes $p, p_1, p_2, \cdots p_r$ and the positive integers $\alpha, \alpha_1, \alpha_2, \cdots \alpha_r$ are such that

$$p^{\alpha} \equiv 1 \pmod 4, \quad p_i^{\alpha_i} \equiv -1 \pmod 4 \quad 1 < i \leq r$$

$m = 1 + p^{\alpha} + p^{2\alpha} + \cdots p^{h\alpha}$ $(h \geq 2)$ is a prime $\equiv 3 \pmod 4$ or a product of twin primes and $q = m+1-4p^{(h-1)\alpha}$ then there exists an Hadamard matrix of order qm. With $p = 5$, $\alpha = 1$, $h = 2$ then $m = 31$, $q = 12$ and thus there exists an H-matrix of order 372.

Additional theorems on the existence of skew-type Hadamard matrices have been published in 1969 by J. Wallis [23, 34]. Also in 1969, W. Kantor [20] and M. Hall, Jr. [17] discuss automorphism groups of Hadamard matrices. During this same year G. Szekeres [31] and D. Blatt and G. Szekeres [7] have published constructions of skew-type Hadamard matrices of order 52 and all orders $m = 2(p^t+1)$, p prime and $p^t \equiv 5 \pmod 8$ giving constructions of skew-type Hadamard matrices of all orders $4m < 92$.

3. <u>SUMMARY</u> At the present moment, this paper , which has considered the historical development of the construction of Hadamard matrices of order $m \leq 200$, still lacks a construction for the Hadamard matrix of order 188.

As has now been shown, there exists different methods of constructing Hadamard matrices for particular orders. Many of the constructed H-matrices in the appendix are a particular type, namely, normalized Hadamard matrices.

Summarizing the problems we see that in Hadamard matrix theory two major unsolved problems remain:

(1) A specific construction for an Hadamard matrix of order $m = 188$ if orders $m \leq 200$ are considered.

(2) Proof of the existence of Hadamard matrices of order $4m = 1,2,4t$ for all values of t.

REFERENCES

[1] Banerjee, K. S., "Some contributions to Hotelling's weighing designs", Sankhyā, 10(1950), 71-82.

[2] Banerjee, K. S., "On the construction of Hadamard matrices", Science and Culture, 14(1949), 434-435.

[3] Banerjee, K. S., "On the variance factors of weighing designs in between two Hadamard matrices", Bull. Cal. Stat. Assoc., 2(1949), 38-42.

[4] Baumert, L. D., Golomb, S. W., and Hall, M., Jr., "Discovery of an Hadamard matrix of order 92", Bull. Amer. Math. Soc., 68(1962), 237-238.

[5] Baumert, L. D. and Hall, M., Jr., "A new construction for Hadamard matrices", Bull. Amer. Math. Soc., 71(1965), 169-170.

[6] Baumert, L. D., "Hadamard matrices of orders 116 and 232", Bull. Amer. Math. Soc., 72(1966), 237.

[7] Blatt, D. and Szekeres, G., "A skew Hadamard matrix of order 52", Con. J. Math., 21(1969), 1319-1323.

[8] Brauer, A., "On a new class of Hadamard determinants", Math. Zeitschrift, 58(1953), 219-225.

[9] Butson, A. T., "Generalized Hadamard matrices", Amer. Math. Soc. Proc., 13(1962), 894-898.

[10] Dade, E. C. and Goldberg, K., "The construction of Hadamard matrices", Michigan Math. J., 6(1959), 247-250.

[11] Dickson, L. E., "Linear Groups with an Exposition of the Galois Field Theory", Leipzig, 1901.

[12] Ehlich, H., "Neue Hadamard-Matrizen", Archiv der Mathematik, 16(1965), 34-36.

[13] Goldberg, K., "Hadamard matrices of order cube plus one", Amer. Math. Soc. Proc., 17(1966), 744-746.

[14] Hadamard, J., "Résolution d'une question relative aux determinants", Bull. des Sciences Mathématiques, (2), 17(1893), Part (1), 240-246.

[15] Hall, M., Jr., "Block designs", Applied Combinatorial Mathematics, Univ. of California, Engineering and Physical Sciences, Extension Series, 369-405.

[16] Hall, M., Jr., "Combinatorial Theory", Ginn and Co., New York.

[17] Hall, M., Jr., "Automorphisms of Hadamard matrices", Siam J. Appl. Math., 17(1969), 1094-1101.

[18] Hotelling, H., "Some improvements in weighing and other experimental techniques", Ann. Math. Stat., 15(1944), 297-306.

[19] Johnsen, E. C., "Integral Solutions to the Incidence equation for finite projective plane cases of orders $n \equiv 2 \pmod 4$, Pacific J. Math., 17 (1966), 97-120.

[20] Kantor, W. M., "Automorphism groups of Hadamard matrices", J. Combinatorial Theory, 6(1969), 279-281.

[21] Kishen, K., "On the design of experiments for weighing and making other types of measurements", Ann. Math. Stat., 16(1945), 294-301.

[22] Mood, A. M., "On Hotelling's weighing problem", Ann. Math. Stat., 17(1946), 432-436.

[23] Paley, R. E. A. C., "On orthogonal matrices", J. Math. and Physics, Massachusetts Inst. of Technology, 12(1933), 311-320.

[24] Plackett, R. L. and Burman, J. P., "The design of optimum multifactorial experiments", Biometrika, 33(1946), 305-325.

[25] Raktoe, B. L. and Federer, W. T., "Characterization of optimal saturated main effects plans of the 2^n factorial", Ann. Math. Stat., 41(1970), 203-206.

[26] Ryser, H. J., "Combinatorial Mathematics", Carus Mathematical Monograph No. 14, The Mathematical Association of America and Wiley, New York.

[27] Scarpis, U., "Sui deteriminanti di valore massimo", Rendiconti della R. Istituto Sombardo di Scienze e Settre, (2), 31(1898), 1441-1446.

[28] Shrikhande, S. S. and Bose, B. C., "A note on a result in the theory of code construction", Information and Control, 2(1959), 183-194.

[29] Spence, E., "A new class of Hadamard matrices", Glasgow Math. J. 18(1967), 59-62.

[30] Sylvester, J. J., "Thoughts on inverse orthogonal matrices, simultaneous sign-successions and tesselated pavements in two or more colours, with applications to Newton's Rule, ornamental tile-work, and the theory of numbers", Phil. Mag. (4), 13(1867), 461-475.

[31] Szekeres, G., "Tournaments and Hadamard matrices", Enseignement Math. 15 (1969), 269-278.

[32] Todd, J. A., "A combinatorial problem", J. Math and Physics, 12(1933), 321-333.

[33] Wallis, J., "A class of Hadamard matrices", J. Combinatorial Theory, 6 (1969), 40-44.

[34] Wallis, J., "A note of a class of Hadamard matrices", J. Combinatorial Theory, 6(1969), 222-223.

[35] Williamson, J., "Hadamard's determinant theorem and the sum of four squares", Duke Math. J., 11(1944), 65-81.

[36] Williamson, J., "Determinants whose elements are 0 and 1", Amer. Math. Monthly, 53(1946), 427-434.

HADAMARD MATRIX OF ORDER M

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \qquad H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$H_{16} = \begin{bmatrix} H_8 & H_8 \\ H_8 & -H_8 \end{bmatrix} \qquad H_{32} = \begin{bmatrix} H_{16} & H_{16} \\ H_{16} & -H_{16} \end{bmatrix} \qquad H_{64} = \begin{bmatrix} H_{32} & H_{32} \\ H_{32} & -H_{32} \end{bmatrix} \qquad H_{128} = \begin{bmatrix} H_{64} & H_{64} \\ H_{64} & -H_{64} \end{bmatrix}$$

$$H_{24} = \begin{bmatrix} H_{12} & H_{12} \\ H_{12} & -H_{12} \end{bmatrix} \qquad H_{40} = \begin{bmatrix} H_{20} & H_{20} \\ H_{20} & -H_{20} \end{bmatrix} \qquad H_{48} = \begin{bmatrix} H_{24} & H_{24} \\ H_{24} & -H_{24} \end{bmatrix} \qquad H_{56} = \begin{bmatrix} H_{28} & H_{28} \\ H_{28} & -H_{28} \end{bmatrix}$$

$$H_{72} = \begin{bmatrix} H_{36} & H_{36} \\ H_{36} & -H_{36} \end{bmatrix} \qquad H_{80} = \begin{bmatrix} H_{40} & H_{40} \\ H_{40} & -H_{40} \end{bmatrix} \qquad H_{88} = \begin{bmatrix} H_{44} & H_{44} \\ H_{44} & -H_{44} \end{bmatrix} \qquad H_{96} = \begin{bmatrix} H_{48} & H_{48} \\ H_{48} & -H_{48} \end{bmatrix}$$

$$H_{104} = \begin{bmatrix} H_{52} & H_{52} \\ H_{52} & -H_{52} \end{bmatrix} \qquad H_{112} = \begin{bmatrix} H_{56} & H_{56} \\ H_{56} & -H_{56} \end{bmatrix} \qquad H_{120} = \begin{bmatrix} H_{60} & H_{60} \\ H_{60} & -H_{60} \end{bmatrix} \qquad H_{136} = \begin{bmatrix} H_{68} & H_{68} \\ H_{68} & -H_{68} \end{bmatrix}$$

$$H_{152} = \begin{bmatrix} H_{76} & H_{76} \\ H_{76} & -H_{76} \end{bmatrix} \qquad H_{160} = \begin{bmatrix} H_{80} & H_{80} \\ H_{80} & -H_{80} \end{bmatrix} \qquad H_{168} = \begin{bmatrix} H_{84} & H_{84} \\ H_{84} & -H_{84} \end{bmatrix} \qquad H_{176} = \begin{bmatrix} H_{88} & H_{88} \\ H_{88} & -H_{88} \end{bmatrix}$$

$$H_{184} = \begin{bmatrix} H_{92} & H_{92} \\ H_{92} & -H_{92} \end{bmatrix} \qquad H_{192} = \begin{bmatrix} H_{96} & H_{96} \\ H_{96} & -H_{96} \end{bmatrix} \qquad H_{200} = \begin{bmatrix} H_{100} & H_{100} \\ H_{100} & -H_{100} \end{bmatrix}$$

HADAMARD MATRIX OF ORDER 12

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1
\end{bmatrix}$$

HADAMARD MATRIX OF ORDER 20

$$
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1
\end{bmatrix}
$$

HADAMARD MATRIX OF ORDER 28

$$
\left[\begin{array}{cccccccccccccccccccccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 \\
1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1
\end{array}\right]
$$

HADAMARD MATRIX OF ORDER 36

1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1
1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1
1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1
1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1
1	1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1
1	-1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1
1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1
1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1
1	1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1
1	-1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1
1	1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
1	-1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1
1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1
1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1
1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1
1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1
1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1
1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1
1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1
1	1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1
1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1
1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1
1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1	-1	1
1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1	1	1
1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	1	-1
1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1	-1	-1
1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1
1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1	1	1
1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1
1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	1	1
1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1	1	-1
1	1	1	1	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	1	-1
1	-1	1	-1	1	-1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	-1

HADAMARD MATRIX OF ORDER 44

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1
1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1
1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1
1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1
1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1
1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1
1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1
1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1
1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1
1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1
1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1
1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1
1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1
1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1
1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1
1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1
1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1
1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1
1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1
1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1
1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1
1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1
1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1
1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1
1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1
1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1
1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1
1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1
1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1
1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1
1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1
1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	1
1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1
1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1
1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1
1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1	-1
1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	1
1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1	-1
1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	1
1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1
1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1
1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1	1
1	1	-1	-1	1	-1	1	-1	-1	1	1	1	-1	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	-1	-1	-1	-1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	1	-1	-1

HADAMARD MATRIX OF ORDER 52

[illegible]

HADAMARD MATRIX OF ORDER 60

1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1	1 1 1 1 1 1
1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-1
1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1-1 1
1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-1
1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-1
1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-1
1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1 1
1 1-1-1-	1 1-1-1 1-	1-1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-1
1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1 1
1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-1
1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1 1
1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1 1-1-1-	1-1 1 1-1 1
1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-1
1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1 1
1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1 1
1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1-1-1-1-	1-1 1-1-1-1
1-1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-1
1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-1
1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1 1
1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-1
1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-1
1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-1
1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1-1-1-1-	1-1 1 1-1-1
1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1 1
1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1 1
1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-1
1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-1
1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-1
1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-1
1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-1
1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1 1
1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1 1
1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1 1
1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1 1
1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1 1
1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-1
1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-1
1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1 1
1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1	1 1-1 1 1 1
1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1 1-1-1	1 1 1-1 1 1
1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-	1 1 1 1-1 1
1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-1
1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1-1 1	1-1 1-1-1	1-1-1 1 1 1
1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-	1 1-1-1 1 1
1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-	1-1-1 1-1 1
1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1-1 1-1	1-1-1 1-1-1
1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1 1 1 1-	1 1 1-1 1-	1 1-1-1-1	1-1-1 1-1	1 1 1 1-1	1-1-1 1-1-1
1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1-1 1	1-1 1-1-1 1
1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-	1 1-1 1-1-1
1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1	1-1 1-1 1-1
1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1	1 1 1-1 1 1
1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1 1-	1-1-1 1-1	1 1 1-1 1 1
1-1 1-1-	1 1-1-1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-	1 1 1-1 1-1
1 1-1 1-	1-1 1-1-1	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1	1-1 1 1 1-1
1-1 1-1	1-1-1 1-1-	1 1 1 1-1	1 1 1 1-1-	1 1 1 1 1	1-1-1-1-1-	1 1 1-1-1-	1-1 1-1-1-	1 1 1-1 1	1-1 1-1 1-	1-1-1 1-1-	1 1-1 1 1 1
1 1-1 1-	1 1-1-1 1-	1-1 1 1 1-	1 1 1 1 1-	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1-1 1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1 1-1 1 1
1 1 1-1	1-1 1-1-1	1-1-1 1 1	1-1 1 1 1	1-1-1 1 1	1 1 1-1-1-	1-1-1 1 1-	1-1-1-1 1-	1-1-1 1 1-	1 1 1-1 1-	1 1-1-1 1-	1-1-1 1-1 1
1 1 1 1-1	1-1 1-1-1	1-1-1 1 1	1 1-1 1 1	1 1-1-1 1	1 1 1 1-1-	1-1-1-1 1	1-1-1-1-1	1-1-1-1 1	1-1 1 1-1	1-1 1-1-1-	1 1-1-1 1-1
1-1 1 1 1	1-1 1-1 1	1-1-1 1-1-	1 1 1-1 1	1 1 1-1-1	1 1 1 1 1-	1-1-1-1-1	1 1-1-1-1-	1 1-1-1-1	1 1-1 1 1-	1 1-1 1-1-	1-1 1-1-1 1
1 1-1 1 1	1 1-1 1-1	1 1-1 1-1	1 1 1 1-1	1-1 1 1 1	1 1-1-1-1-	1-1 1 1-1-	1-1-1 1-1-	1 1-1 1-1	1 1-1 1-1	1-1-1-1 1-	1-1-1 1-1-1

HADAMARD MATRIX OF ORDER 68

HADAMARD MATRIX OF ORDER 76

HADAMARD MATRIX OF ORDER 64

[illegible]

HADAMARD MATRIX OF ORDER 92

$$\text{where } H_{92} = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix}$$

$$A = \begin{pmatrix}
1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
-1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\
-1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
-1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
-1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 \\
-1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
-1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\
-1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
-1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1
\end{pmatrix}$$

$$B = \left(\begin{array}{ccccccccccccccccccccccc}
1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 \\
-1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\
\hline
1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
-1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\
\hline
1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\
-1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
-1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & \text{[illegible]} & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & -1 & 1
\end{array}\right)$$

$$
C = \begin{pmatrix}
1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
-1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\
-1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 \\
-1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 \\
-1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1
\end{pmatrix}
$$

$$
D = \left(\begin{array}{rrr|rrr|rrr|rrrr|rrr|rrr|rrrr}
1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\
-1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 \\
\hline
-1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\
\hline
-1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & -1 \\
-1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & 1 & 1
\end{array}\right)
$$

HADAMARD MATRIX OF ORDER 100

$$\text{where } H_{100} = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix}$$

$$A = \begin{bmatrix}
1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1
\end{bmatrix}$$

$$B = \begin{bmatrix}
1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
-1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1
\end{bmatrix}$$

$$
C = \left[\begin{array}{rrrr|rrrrr|rrrrr|rrrrr|rrrrr|r}
1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 & 1 & 1
\end{array}\right]
$$

$$
D = \left[\begin{array}{rrrr|rrrrr|rrrrr|rrrrr|rrrrr|r}
1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1
\end{array}\right]
$$

HADAMARD MATRIX OF ORDER 108

HADAMARD MATRIX OF ORDER 116

$$\text{where } H_{116} = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix}$$

$A =$

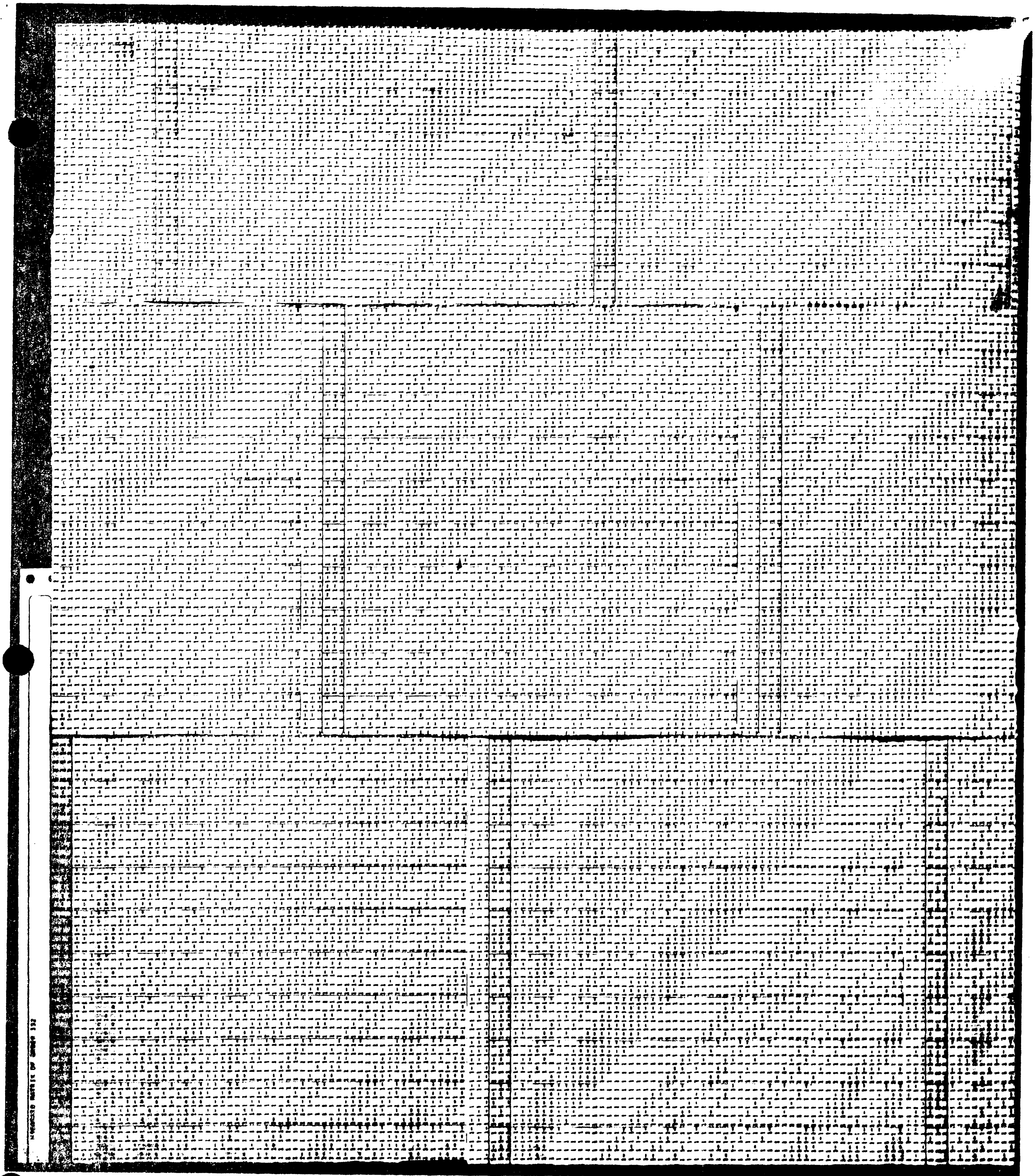
```
 1 1-1-1 | 1-1-1 1-1 | 1 1 1-1 1 | 1 1 1-1 1 | 1 1-1 1 1 | [illegible]
 1 1 1-1-| 1 1-1-1 1-| 1 1 1 1-1 | 1 1 1 1-1 | 1 1 1-1 1-| 1-1 1-1-1
-1 1 1 1-| 1-1 1-1-1 | 1-1 1 1 1-| 1 1 1 1 1-| 1 1 1 1-1 | 1-1-1 1-1
-1-1 1 1 | 1-1-1 1-1-| 1 1-1 1 1 | 1-1 1 1 1 | 1-1 1 1 1-| 1 1-1-1 1
 1-1-1 1 | 1 1-1-1 1-| 1-1 1-1 1 | 1 1-1 1 1 | 1 1-1 1 1 | 1-1 1-1-1
-1 1-1-1 1 1 1-1-1 1-1-1 1-1 1 1 1-1 1 1 1 1-1 1 1 1-1 1-1
-1-1 1-1-1 1 1 1-1-1 1-1-1 1-1 1 1 1-1 1 1 1 1-1 1 1 1-1 1
 1-1-1 1-1-1 1 1 1-1-1 1-1-1 1-1 1 1 1-1 1 1 1 1-1 1 1 1-1
-1 1-1-1 | 1-1-1 1 1 | 1-1-1 1-1-| 1 1-1 1 1 | 1-1 1 1 1 | 1-1 1 1 1
 1-1 1-1-| 1 1-1-1 1 | 1 1-1-1 1-| 1-1 1-1 1 | 1 1-1 1 1 | 1 1-1 1 1
 1 1-1 1-| 1-1 1-1-1 | 1 1 1-1-1 | 1-1-1 1-1 | 1 1 1-1 1 | 1 1 1-1 1
 1 1 1-1 | 1-1-1 1-1-| 1 1 1 1-1-| 1 1-1-1 1-| 1 1 1 1-1 | 1 1 1 1-1
-1 1 1 1-| 1 1-1-1 1-| 1-1 1 1 1-| 1-1 1-1-1 | 1-1 1 1 1-| 1 1 1 1 1
 1-1 1 1 | 1-1 1-1-1 | 1-1-1 1 1 | 1-1-1 1-1-| 1 1-1 1 1 | 1-1 1 1 1
 1 1-1 1 | 1 1-1 1-1-| 1 1-1-1 1 | 1 1-1-1 1-| 1-1 1-1 1 | 1 1-1 1 1
 1 1 1-1 | 1 1 1-1 1-| 1-1 1-1-1 | 1 1 1-1-1 | 1-1-1 1-1 | 1 1 1-1 1
 1 1 1 1-| 1 1 1 1-1 | 1-1-1 1-1-| 1 1 1 1-1-| 1 1-1-1 1-| 1 1 1 1-1
-1 1 1 1 | 1-1 1 1 1-| 1 1-1-1 1-| 1-1 1 1 1-| 1-1 1-1-1 | 1-1 1 1 1
 1-1 1 1 | 1 1-1 1 1 | 1-1 1-1-1 | 1-1-1 1 1 | 1-1-1 1-1-| 1 1-1 1 1
 1 1-1 1 | 1 1 1-1 1 | 1 1-1 1-1-| 1 1-1-1 1 | 1 1-1-1 1-| 1-1 1-1 1
 1 1 1-1 | 1 1 1 1-1 | 1 1 1-1 1-| 1-1 1-1-1 | 1 1 1-1-1 | 1-1-1 1-1
-1 1 1 1-| 1 1 1 1 1-| 1 1 1 1-1 | 1-1-1 1-1-| 1 1 1 1-1-| 1 1-1-1 1
 1-1 1 1 | 1-1 1 1 1 | 1-1 1 1 1-| 1 1-1-1 1-| 1-1 1 1 1-| 1-1 1-1-1
-1 1-1 1 | 1 1-1 1 1 | 1 1-1 1 1 | 1-1 1-1-1 | 1-1-1 1 1 | 1-1-1 1-1
-1-1 1-1 | 1 1 1-1 1 | 1 1 1-1 1 | 1 1-1 1-1-| 1 1-1-1 1 | 1 1-1-1 1
 1-1-1 1-| 1 1 1 1-1 | 1 1 1 1-1 | 1 1 1-1 1-| 1-1 1-1-1 | 1 1 1-1-1
-1 1-1-1 | 1-1 1 1 1-| 1 1 1 1 1-| 1 1 1 1-1 | 1-1-1 1-1-| 1 1 1 1-1
-1-1 1-1-| 1 1-1 1 1 | 1-1 1 1 1 | 1-1 1 1 1-| 1 1-1-1 1-| 1-1 1 1 1
 1-1-1 1-| 1-1 1-1 1 | 1 1-1 1 1 | 1 1-1 1 1 | 1-1 1-1-1 | 1-1-1 1 1
```

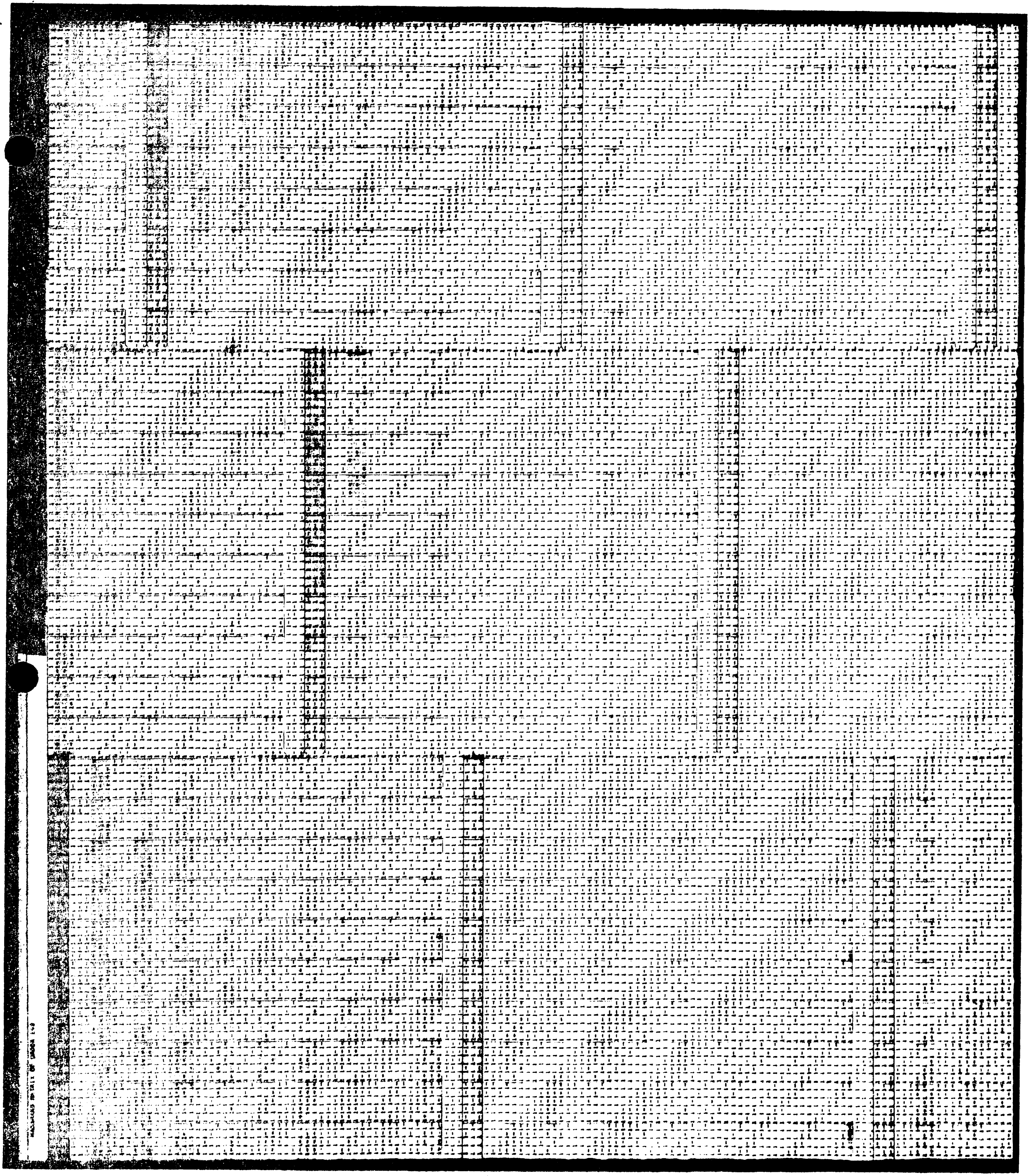
$B =$

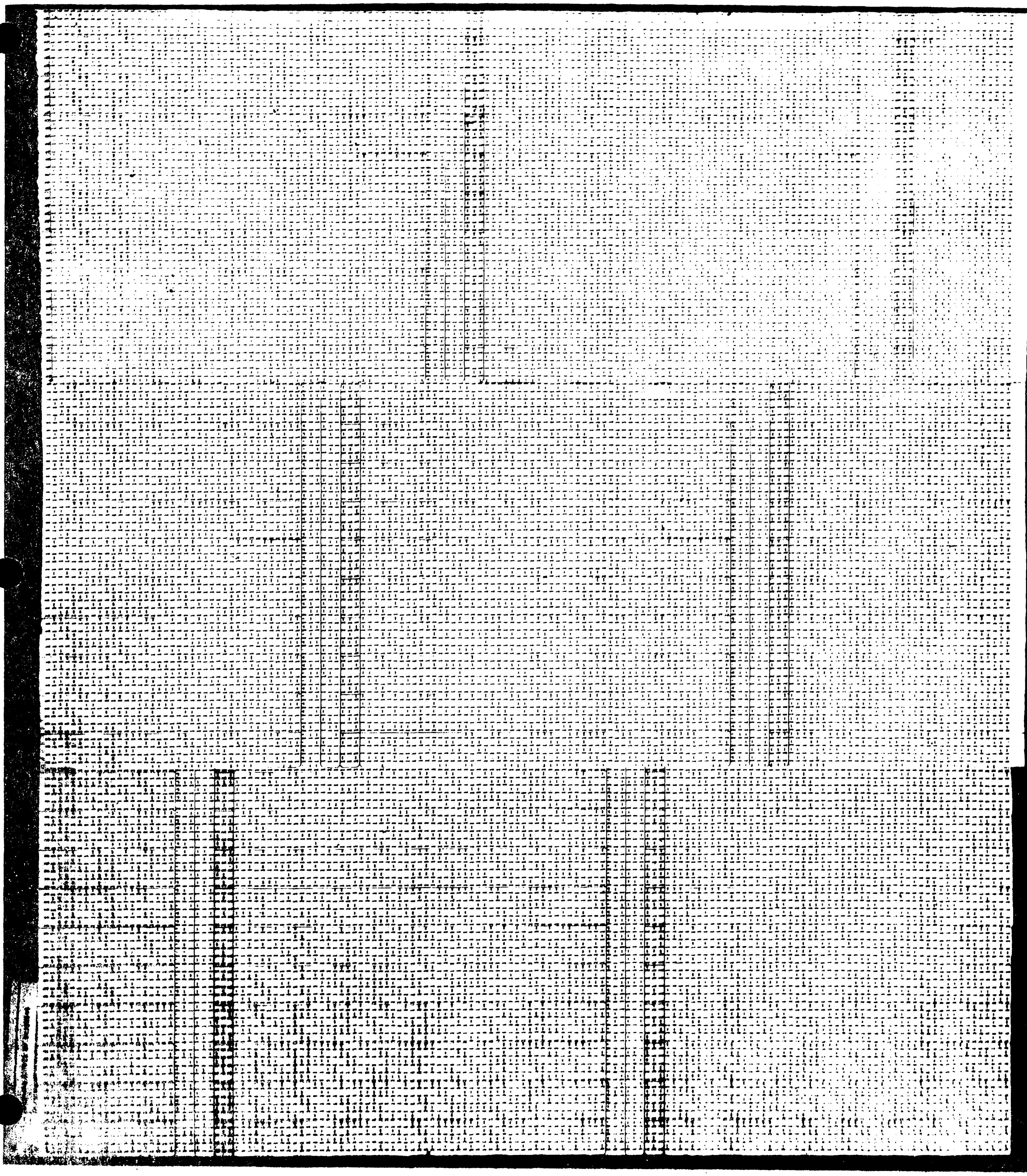
```
 1 1 1 1-| 1 1 1-1 1-| 1-1-1 1 1 | 1 1 1 1-1-| 1-1 1-1 1 | 1-1 1 1 1
 1 1 1 1 | 1-1 1 1-1 | 1-1-1-1 1 | 1 1 1 1 1-| 1-1-1 1-1 | 1 1-1 1 1
 1 1 1 1 | 1 1-1 1 1-| 1 1-1-1-1 | 1 1 1 1 1 | 1-1-1-1 1-| 1 1 1-1 1
 1 1 1 1 | 1 1 1-1 1 | 1-1 1-1-1-| 1 1 1 1 1 | 1 1-1-1-1 | 1-1 1 1-1
-1 1 1 1 | 1 1 1 1-1 | 1 1-1 1-1-| 1-1 1 1 1 | 1 1 1-1-1-| 1 1-1 1 1
 1-1 1 1 | 1 1 1 1 1-| 1 1 1-1 1-| 1-1-1 1 1 | 1 1 1 1-1-| 1-1 1-1 1
 1 1-1 1 | 1 1 1 1 1 | 1-1 1 1-1 | 1-1-1-1 1 | 1 1 1 1 1-| 1-1-1 1-1
-1 1 1-1 | 1 1 1 1 1 | 1 1-1 1 1-| 1 1-1-1-1 | 1 1 1 1 1 | 1-1-1-1 1
 1-1 1 1-| 1 1 1 1 1 | 1 1 1-1 1 | 1-1 1-1-1-| 1 1 1 1 1 | 1 1-1-1-1
-1 1-1 1 | 1-1 1 1 1 | 1 1 1 1-1 | 1 1-1 1-1-| 1-1 1 1 1 | 1 1 1-1-1
-1-1 1-1 | 1 1-1 1 1 | 1 1 1 1 1-| 1 1 1-1 1-| 1-1-1 1 1 | 1 1 1 1-1
-1-1-1 1-| 1 1 1-1 1 | 1 1 1 1 1 | 1-1 1 1-1 | 1-1-1-1 1 | 1 1 1 1 1
 1-1-1-1 | 1-1 1 1-1 | 1 1 1 1 1 | 1 1-1 1 1-| 1 1-1-1-1 | 1 1 1 1 1
 1 1-1-1-| 1 1-1 1 1-| 1 1 1 1 1 | 1 1 1-1 1 | 1-1 1-1-1-| 1 1 1 1 1
 1 1 1-1-| 1-1 1-1 1 | 1-1 1 1 1 | 1 1 1 1-1 | 1 1-1 1-1-| 1-1 1 1 1
 1 1 1 1-| 1-1-1 1-1 | 1 1-1 1 1 | 1 1 1 1 1-| 1 1 1-1 1-| 1-1-1 1 1
 1 1 1 1 | 1-1-1-1 1-| 1 1 1-1 1 | 1 1 1 1 1 | 1-1 1 1-1 | 1-1-1-1 1
 1 1 1 1 | 1 1-1-1-1 | 1-1 1 1-1 | 1 1 1 1 1 | 1 1-1 1 1-| 1 1-1-1-1
-1 1 1 1 | 1 1 1-1-1-| 1 1-1 1 1-| 1 1 1 1 1 | 1 1 1-1 1 | 1-1 1-1-1
-1-1 1 1 | 1 1 1 1-1-| 1-1 1-1 1 | 1-1 1 1 1 | 1 1 1 1-1 | 1 1-1 1-1
-1-1-1 1 | 1 1 1 1 1-| 1-1-1 1-1 | 1 1-1 1 1 | 1 1 1 1 1-| 1 1 1-1 1
 1-1-1-1 | 1 1 1 1 1 | 1-1-1-1 1-| 1 1 1-1 1 | 1 1 1 1 1 | 1-1 1 1-1
-1 1-1-1-| 1 1 1 1 1 | 1 1-1-1-1 | 1-1 1 1-1 | 1 1 1 1 1 | 1 1-1 1 1
 1-1 1-1-| 1-1 1 1 1 | 1 1 1-1-1-| 1 1-1 1 1-| 1 1 1 1 1 | 1 1 1-1 1
 1 1-1 1-| 1-1-1 1 1 | 1 1 1 1-1-| 1-1 1-1 1 | 1-1 1 1 1 | 1 1 1 1-1
-1 1 1-1 | 1-1-1-1 1 | 1 1 1 1 1-| 1-1-1 1-1 | 1 1-1 1 1 | 1 1 1 1 1
 1-1 1 1-| 1 1-1-1-1 | 1 1 1 1 1 | 1-1-1-1 1-| 1 1 1-1 1 | 1 1 1 1 1
 1 1-1 1 | 1-1 1-1-1-| 1 1 1 1 1 | 1 1-1-1-1 | 1-1 1 1-1 | 1 1 1 1 1
 1 1 1-1 | 1 1-1 1-1-| 1-1 1 1 1 | 1 1 1-1-1-| 1 1-1 1 1-| 1 1 1 1 1
```

$$
C=\left(\begin{array}{rrrr|rrrrr|rrrrr|rrrrr|rrrrr|rrrrr}
1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
\hline
-1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \\
\hline
1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\
-1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 \\
-1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1
\end{array}\right)
$$

$$
D=\left(\begin{array}{rrrr|rrrrr|rrrrr|rrrrr|rrrrr|rrrrr}
1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\
-1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\
-1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 \\
1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 & -1 \\
-1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1
\end{array}\right)
$$







HADAMARD MATRIX OF ORDER 156

$$
\text{where } H_{156} = \begin{pmatrix}
A & A & A & B & -B & C & -C & -D & B & C & -D & -D \\
A & -A & B & -A & -B & -D & D & -C & -B & -D & -C & -C \\
A & -B & -A & A & -D & D & -B & B & -C & -D & C & -C \\
B & A & -A & -A & D & D & D & C & C & -B & -B & -C \\
B & -D & D & D & A & A & A & C & -C & B & -C & B \\
B & C & -D & D & A & -A & C & -A & -D & C & B & -B \\
D & -C & B & -B & A & -C & -A & A & B & C & D & -D \\
-C & -D & -C & -D & C & A & -A & -A & -D & B & -B & -B \\
D & -C & -B & -B & -B & C & C & -D & A & A & A & D \\
-D & -B & C & C & C & B & B & -D & A & -A & D & -A \\
C & -B & -C & C & D & -B & -D & -B & A & -D & -A & A \\
-C & -D & -D & C & -C & -B & B & B & D & A & -A & -A
\end{pmatrix}
$$

$$
A = \begin{bmatrix}
1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\
-1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\
-1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
-1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 \\
1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 & -1 \\
-1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 1 \\
1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\
-1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 \\
-1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\
-1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
-1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\
-1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1
\end{bmatrix}
$$

$$C = \begin{bmatrix}
1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 \\
-1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 \\
1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\
-1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & -1 \\
-1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 \\
-1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & 1 & 1
\end{bmatrix}$$

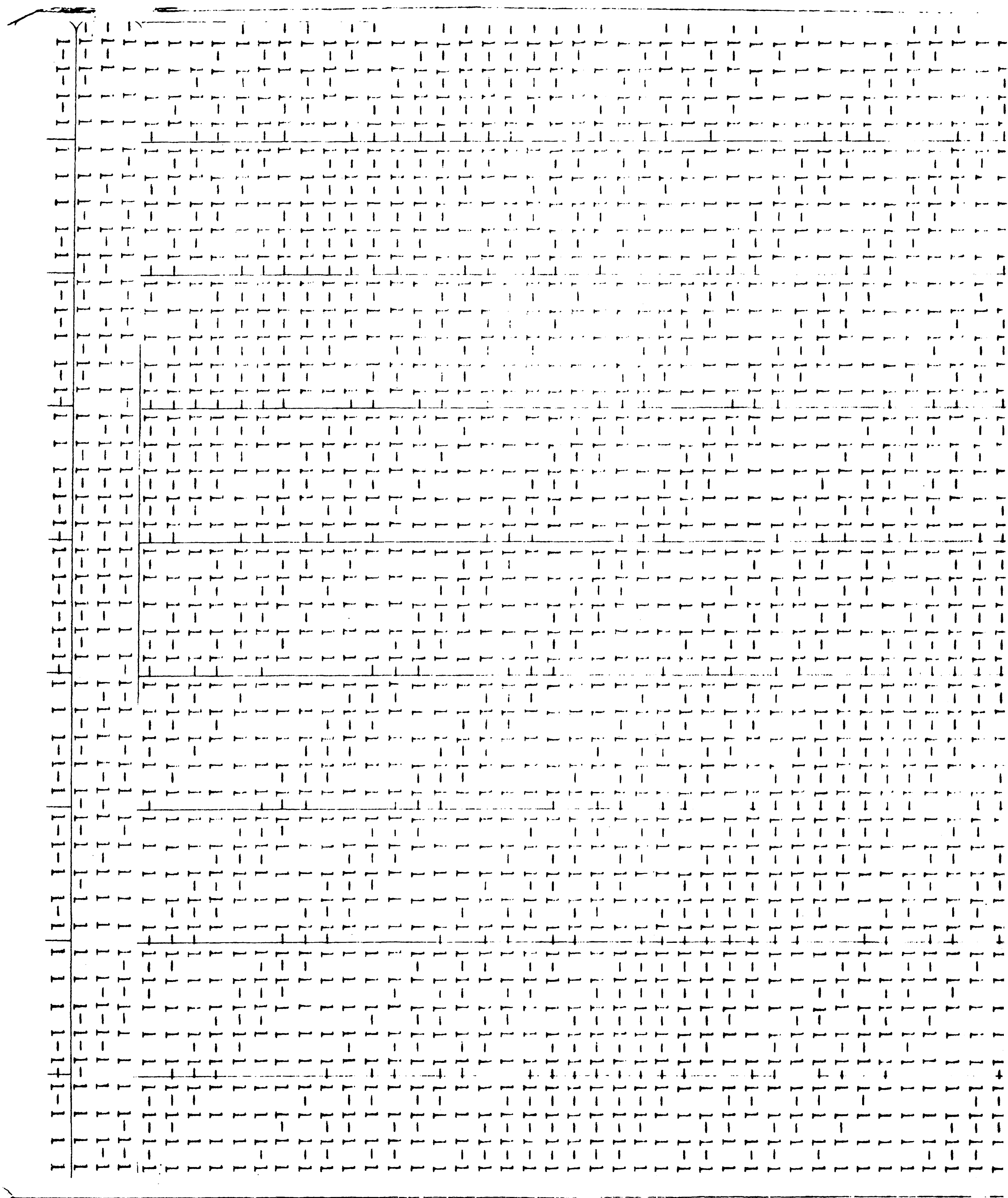
$$D = \begin{bmatrix}
1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
-1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 \\
-1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 \\
-1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 \\
-1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & 1
\end{bmatrix}$$

HADAMARD MATRIX OF ORDER 172

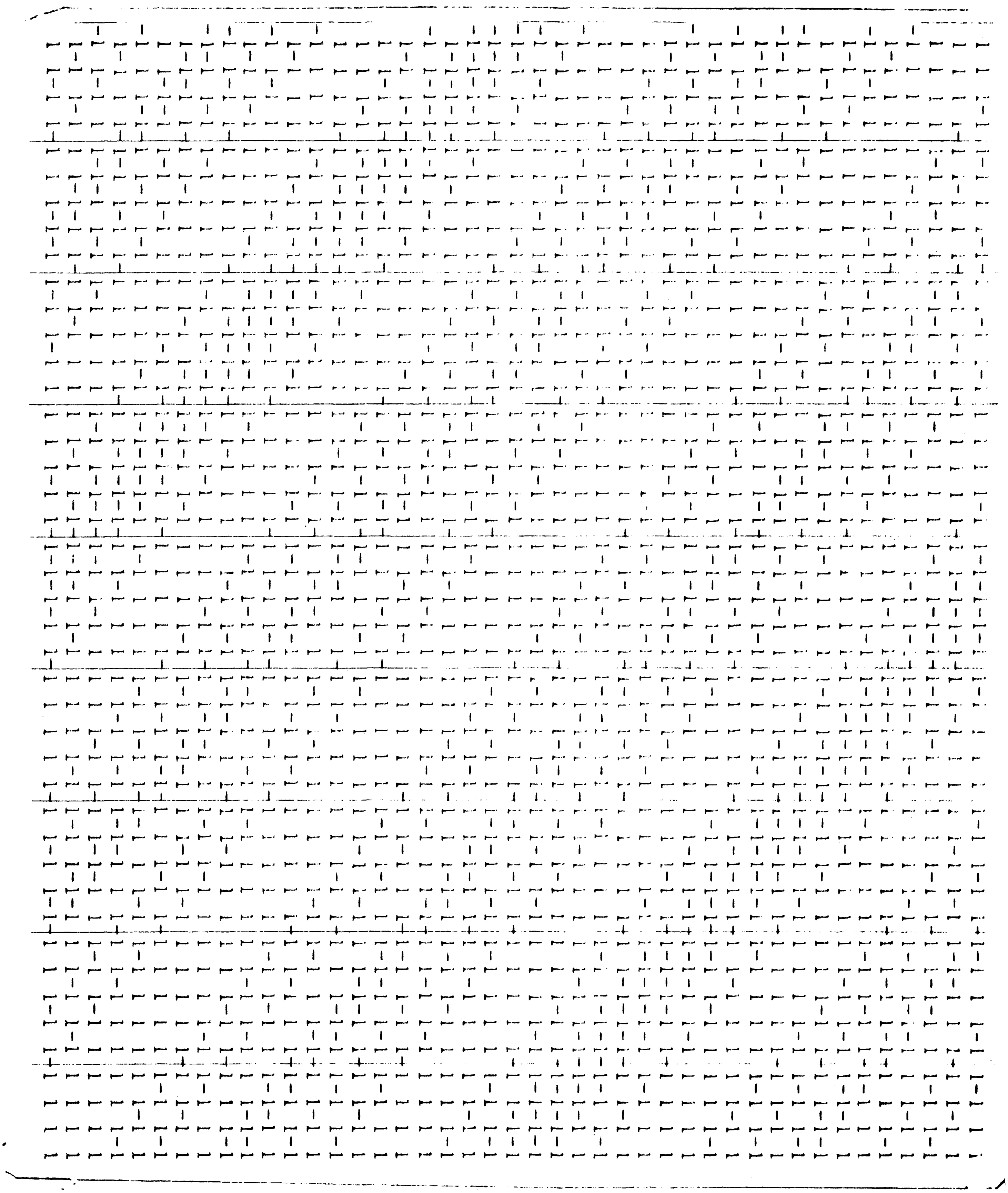
$$\text{where } H_{172} = \begin{bmatrix} A & B & C & D \\ -B & A & -D & C \\ -C & D & A & -B \\ -D & -C & B & A \end{bmatrix}$$

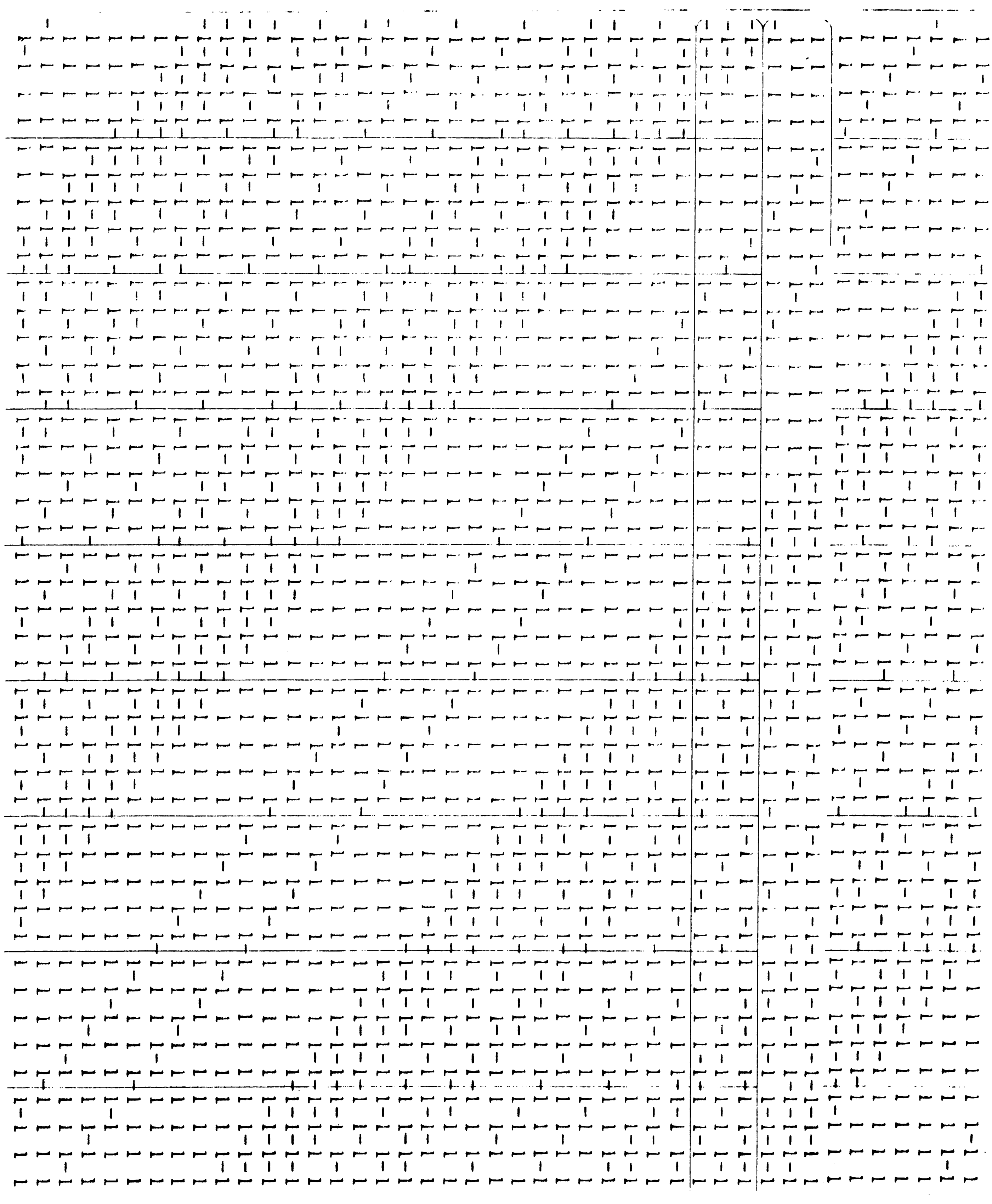
D

11



11





D =

1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1
-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1
-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1
-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1
1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1
1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1
-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1
-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1
1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1
1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1
1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1
1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1
-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1
1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1
-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1
1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1
1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1
1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1
-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1
1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1
1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1
-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1
-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1
1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1
1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1
-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1
1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1
1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1
1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1
-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1
1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1
-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	1
1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1
1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1	1
1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1	1
1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1	-1
-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	-1
-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1
1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1
1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1
-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1	-1
-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	1	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	1	-1	-1	-1	1	-1
-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	1	-1	1	1	1	-1	[illegible]	1	-1	-1	1	1	-1	1	1	1	-1	1	-1	1	1	1	1	-1	-1	1	[illegible]	[illegible]	-1	-1	1